A finite element analysis for asymmetric contraction after coiling of hot-rolled steel

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1. Introduction

A hot-rolled steel strip is generally stocked in the form of a hollow cylindrical coil after the hot rolling process. The hot coil is cooled from 500–700 °C to room temperature over a 4–5-day period under natural air cooling conditions as presented by Park et al. (1998) and Saboonchi and Hassanpour (2007). In most hot strip rolling processes, the phase transformation of steel was finished on the run-out table (ROT) before coiling (Han et al., 2002), and the hot coil is normally cooled down but maintaining the cylindrical shape (Park et al., 2009). However, asymmetric contraction occurs in an actual mill during cooling after the coiling of hot-rolled steel, which has significantly large hardenability due to high content of carbon or other alloying elements and shows incomplete phase transformation prior to coiling, as shown in Fig. 1. This asymmetric contraction behavior is closely related to the phase transformation that occurs after cooling, and cannot be described by conventional creep behavior. This shape change in the hot coil causes acute problems in industrial applications, such as serious scratching on the strip surface during uncoiling.

In this study, the concept of the transformation plasticity was adopted to describe this asymmetric contraction behavior occurring during the phase transformation in hot-rolled steel after coiling. The transformation plasticity refers to the permanent deformation that occurs during a phase transformation of ferrous or other alloys under an applied stress, which is even lower than the yield stress of the material (Greenwood and Johnson, 1965; Han and Lee, 2002). Owing to its technological importance, several theoretical models based on continuum mechanics have been proposed to quantify the transformation plasticity phenomenon, one widely accepted description being the contribution of Greenwood and Johnson (1965). In comparison with the previous continuum-based theory, approaches, which consider the microstructural aspects of phase transformation, have also been suggested. Han and Suh (2003) and Han et al. (2004) developed a microstructure-based model using the Kurdjumov–Sachs (KS) orientation relationship between fcc and bcc to interpret the transformation plasticity phenomenon during the displacive transformation under an applied external stress. Recently, Han et al. (2007, 2008) suggested a constitutive equation for the transformation plasticity based on the diffusion mechanism of the migrating interface during the diffusional phase transformation, which can be described as accelerated Coble creep. The constitutive equation for transformation plasticity was incorporated into a general purpose implicit finite element (FE) program. In addition to thermo-elasto-plastic constitutive equations, the phase transformation kinetics was characterized by a Johnson–Mehl–Avrami–Kolmogorov (JMAK) type equation. The validity of the proposed model was examined by reproducing the asymmetric contraction behavior of the coil. The effect of some selected process variables on the asymmetric
contraction was investigated through a series of process simulations.

2. Model development

2.1. Phase transformation kinetics

High carbon steel (0.82 wt.%C–0.18 wt.%Si–0.40 wt.%Mn–0.15 wt.%Cr), which has a sufficient hardenability to retain a large amount of untransformed austenite at the stage of coiling in hot rolling process, was used. The untransformed austenite in the hot-rolled coil transformed to pearlite during air cooling due to the high carbon content of the steel. The transformation kinetics for the austenite-to-pearlite transformation was characterized using a carbon content of the steel. The transformation kinetics for the coil transformed to pearlite during air cooling due to the high process, was used. The untransformed austenite in the hot-rolled untransformed austenite at the stage of coiling in hot rolling which has a sufficient hardenability to retain a large amount of

tions.

Fig. 1. An example of asymmetric contraction of hot coil after coiling.

Table 1

| k and n values of Eq. (2) for austenite-to-pearlite transformation of high carbon steel (0.82 wt.%C–0.18 wt.%Si–0.40 wt.%Mn–0.15 wt.%Cr) |
|---|---|
| ln(k) | n |
| 83.1859 * 12.5562 ln [sin((6.2832T/|4(Teq - 80)|)] + 1.3172 ln(AGS) + (7.5854 - 1.9038 [Xc] C) | 7.6049 - 5.2261 [Xc] |

where \( X_i \) follows:

\[
X = 1 - \exp(-k t^n)
\]

where \( X \) is the transformed phase fraction, \( t \) is the total time for the transformation at a given temperature, respectively, \( n \) is the time exponent and \( k \) is the parameter depending on the temperature and transformation mechanism.

In order to extend the JMAK equation for the austenite-to-pearlite transformation into the non-isothermal process, the concept of additivity rule was introduced under the assumption that the cooling curve can be divided into small time intervals within which the kinetics parameters in Eq. (1) are constant (Sheil, 1935). The transformed phase fraction until the \( i \)-th step, \( X_i \), could be expressed as follows:

\[
X_i = 1 - \exp(-X_i^{ex} + \Delta X_i^{ex}), \quad \Delta X_i^{ex} = n k t_i^{eq(n-1)} \Delta t,
\]

where \( t_i^{eq} \) is the equivalent transformation time needed to transform into the fraction of \( X_i^{ex} \) at the temperature of the \( i \)-th step, and \( \Delta t \) is the time step corresponding to the \( i \)-th step. The constants of Eq. (2), \( k \) and \( n \), were determined by applying an inverse additivity technique into the dilatation data obtained from the continuous heat treatment tests (Han and Park, 2001). Table 1 lists the \( k \) and \( n \) values of the austenite-to-pearlite transformation for high carbon steel. Here, \( C_y \) is the carbon content in untransformed austenite, AGS (\( \mu \text{m} \)) means the prior austenite grain size and \( T_{eq} \) (K) is the equilibrium transformation temperature. \( D_c \) is the composition and temperature dependent diffusivity of carbon in austenite, and the equation assessed by Årgen (1986) was used at a given temperature as follows:

\[
D_c = 4.53 \times 10^{-7} \left\{ 1 + y_c(1 - y_c - \frac{8339.9}{T}) \right\} \times \exp \left( -\left( \frac{1}{T} - 2.221 \times 10^{-4} \right) (17767 - y_c 26436) \right)
\]

where \( y_c \) depends on the mole fraction of carbon, \( x_c \), as \( y_c = x_c/(1 - x_c) \). The values for \( C_y \) and \( T_{eq} \) were calculated by thermodynamic analysis using Thermo-Calc (Sundman et al., 1985).

2.2. Heat transfer

In the thermo-mechanical analysis involving the phase transformation, consideration of the temperature increase caused by the latent heat generated by enthalpy changes during the phase transformation is quite important. The isotropic heat transfer equation is represented as follows:

\[
\rho C_p \dot{T} = \nabla \cdot (k \nabla T) + \Delta H \cdot \dot{X}
\]

where \( \rho \), \( C_p \) and \( k \) are the density, heat capacity and thermal conductivity, respectively. \( \Delta H \) and \( \dot{X} \) are the heat evolution due to the phase transformation and transformation rate, respectively. Here, \( C_p \), \( \Delta H \) and \( \dot{X} \) were obtained from thermodynamic and phase transformation analyses. \( \rho \) and \( k \) were determined as a function of temperature and the chemical composition of the steel using Miettinen’s data (Miettinen, 1995). The latent heating by the phase transformation was added to the regular thermal constitutive behavior via a user subroutine defining a thermal behavior of a material, UMATHT in ABAQUS (ABAQUS Inc., 2006). The user subroutine, UMATHT, was used with a fully integrated coupled temperature-displacement and heat transfer element. In the user subroutine, the variables to be updated were the internal thermal energy per unit mass, the heat flux and their variations with respect to temperature, and the change in the heat flux with respect to the spatial gradient of temperature.

2.3. Transformation plasticity

The transformation plasticity proposed by Han et al. (2007) was adopted. This model is based on the diffusion mechanism of the migrating interface during the diffusional phase transformation. According to this theory, it was assumed that the overall atomic flux across the phase interface is perpendicular to the interface, and that the migrating atoms are relocated to the nearest atomic sites in the transformed phase. However, when a stress is applied, the migrating atoms may move to positions where they can release the applied stress field, which induces an atomic flux along the phase interface.

Using this concept, Han et al. (2007) derived a constitutive equation describing the transformation plasticity strain rate \( \dot{\varepsilon}^{TP} \) as a function of the transformation rate, temperature and applied stress.
as follows:
\[
\dot{\varepsilon}^{TP} = \frac{1}{3} \frac{\sigma \Omega}{k_B T} c_0 \exp \left( \frac{Q_f}{k_B T} \right)
\]
(5)
where \(d_0\), \(\delta\), and \(\Omega\) represent the initial grain size of the parent phase, the effective thickness of the interface and the volume of the vacancy, respectively. The Boltzmann constant, \(k_B\), has a value of 1.38 \times 10^{-23} \text{J/K}. \(c_0\) is a dimensionless constant determined by the change in thermal entropy associated with the formation of vacancies and \(Q_f\) is the formation enthalpy of the vacancy at the interface. \(\sigma\) and \(\dot{X}\) are the applied stress and transformation rate, respectively.

It was reported that the transformation strain calculated by the model agrees well with the experimental measurements during the transformation of steels under a variety of uniaxial compressive stresses (Han et al., 2007). In addition, this model had also been used successfully to simulate the thermally activated behavior of transformation strain.

In this study, the initial grain size of the austenite phase of high carbon steel was found to be 30 \(\mu\)m. The parameters in Eq. (5) of the vacancy volume (\(\Omega\)), the effective thickness of the interface (\(\delta\)) and the formation enthalpy of the vacancy at the interface (\(Q_f\)) were 1.21 \times 10^{-23} \text{m}^3 (Frost and Ashby, 1982), 1 nm and 80 kJ/mol (Han et al., 2007), respectively. The constant \(c_0\) was adjusted using the constrained Rosenbrock technique as an optimization procedure (Kuester and Mize, 1973), which was performed while changing the constants systematically until the sum of the squared differences between the experimental and calculated data reached a minimum.

### 2.4. Constitutive formulations

The stress increment of Cauchy stress, \(d\sigma\), is
\[
d\sigma = C^e : ds^e
\]
(6)
where \(C^e\) and \(ds^e\) are the elastic stiffness tensor and elastic strain increment, respectively. The total strain increment, \(d\varepsilon^t\), is
\[
d\varepsilon^t = ds^e + ds^p + ds^{TP} + ds^{TP}
\]
(7)
where \(ds^e, ds^{TP}\) and \(ds^p\) are the volumetric strain increment due to the phase transformation and temperature change, the transformation plasticity strain increment associated with the phase transformation and the plastic strain increment, respectively. The volumetric strain increment by the phase transformation was assumed to be a linear mixture rule of strain increments of existing phases. Therefore,
\[
ds^e = X_p ds^e_p + X_o ds^e_o = \left[ X_p \left( \frac{1}{3} \frac{\rho_p}{\rho} \right) + X_o \left( \frac{1}{3} \frac{\rho_o}{\rho} \right) \right] I
\]
(8)
where the subscripts \(p\) and \(o\) are austenite and pearlite, respectively, and \(I\) indicates the identity tensor. The density of each phase was determined as a function of temperature to consider the effect of both phase transformation and temperature change. Here, the plastic strain increment was ignored because the stress that developed due to gravity is sufficiently small. Therefore, the stress increment becomes:
\[
d\sigma = C^e : (d\varepsilon^{e} - d\varepsilon^{TP})
\]
(9)

### Table 2
Elastic modulus of high carbon steel at various temperatures (Wray, 1980).

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>25</th>
<th>300</th>
<th>600</th>
<th>700</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>171</td>
<td>163</td>
<td>144</td>
<td>131</td>
<td>107</td>
</tr>
</tbody>
</table>

The temperature dependent isotropic elastic modulus was used to obtain the elastic stiffness of high carbon steel, as listed in Table 2 (Wray, 1980; Koric and Thomas, 2008). The Poisson’s ratio of the steel was assumed to be 0.3 over the entire temperature range. The constitutive models were incorporated into the user material subroutine, UMAT of ABAQUS/Standard (AB AQUS Inc., 2006).

### 2.5. Calculation procedure

In order to model the entire system of a hot-rolled coil, a large number of elements are needed for the FE calculation because the coil is not in bulk form but is a repeated laminate structure of a thin sheet. In this study, the behavior of one layer of thin sheet was analyzed through the FE simulation to calculate the effect of the transformation plasticity on the asymmetric contraction of the hot-rolled coil.

Before the single layer calculation, a bulk calculation, assuming that the unit layer of the hot-rolled coil consists of steel, oxide and an interface (Fig. 2), was performed to determine the temperature and stress field in the coil as a function of time. The distribution and the history of temperature and stress were used in the FE calculation. In Fig. 2, the interface indicates the layer, which is composed of contact points between the sheets and a void filled with air. For the FE simulation of the bulk coil, the mesh system was designed three-dimensionally by considering the symmetry, as shown in Fig. 3. The inner and outer radii and the height of the coil were 762 mm, 1918 mm and 619 mm, respectively. The coil was constructed with 8000 elements, and the side surface part of the coil was divided into finer elements because large temperature variations developed at this region during cooling. The gravity force was applied to the coil, as shown in Fig. 3. The heat transfer coefficient at the coil surface was determined by minimizing the difference between the measured and calculated temperature profiles on the coil surface during cooling.

Orthotropic material properties based on the frame of the coil geometry were used in the bulk simulation by considering the material anisotropy, as shown in Fig. 2. The elastic modulus along the radial direction was expressed as follows (Courtney, 2000):
\[
E_r = \frac{(t_r + t_0 + t_i)E_sE_p}{t_rE_sE_p + t_0E_sE_0 + t_iE_sE_i}
\]
(10)
where \(E_s\) is the elastic modulus of the bulk coil along the radial direction. \(E_r, E_p\) and \(E_s\) are the elastic modulus and \(t_r, t_0, t_i\) are...
the thickness of the sheet steel, oxide and interface layer, respectively. The elastic modulus of the steel listed in Table 2 was used. The elastic moduli of the oxide and interface layer were 200 GPa (Morrel, 1987) and $10^{-4}$ GPa, respectively, and the thickness of the steel sheet was 3.3 mm. The thicknesses of the oxide and interface layer were set to 7 μm and 26 μm, respectively (Park et al., 1998). The circumferential elastic modulus was used as the value of the sheet steel listed in Table 2. Regarding the thermal conductivity, the radial directional property was assumed to be one tenth that of the circumferential value based on a consideration of the oxide and interface layer (Park et al., 1998). The circumferential thermal conductivity was used as the value for high carbon steel (Miettien, 1995).

Since the stiffness of the layered material is proportional to the layered material thickness (Bickford, 1993; Hua et al., 2009), the stiffness in the above bulk calculation was overestimated compared to that of the real coil, which consists of repeated layers of thin sheets. Therefore, the level of asymmetric contraction of the hot-rolled steel due to gravity in the bulk calculation should be underestimated. Here, only a single inner layer of thin sheet was analyzed through the FE simulation in order to simulate the influence of the transformation plasticity upon an asymmetric contraction of the coil during cooling. Fig. 4 shows a FE mesh of single layer in the coil. The distribution and history of temperature and stress, which were obtained from the bulk calculation, was used in the calculation for the single layered sheet. This modeling may result in some exaggeration of the results because the effect of other layers was excluded. However, the single layer modeling can be an effective and helpful method of revealing the origin of the asymmetric contraction of the hot-rolled steel, and confirming the effect of some selected process variables, such as the coiling temperature, amount of untransformed austenite, layer thickness of the coil, and length of the coil.

3. Experimental

Dilatometric specimens of the high carbon steel of the cylindrical type, 3 mm in diameter and 10 mm in length, were prepared. The specimens were austenitized at 900 °C for 10 min in a vacuum and cooled to room temperature at various cooling rates ranging from 0.5 to 20 °C/s. The dilatometric measurements were carried out using a transformation dilatometer (Dilatronic III, Theta Inc.), which heats the specimen with an induction coil and detects the change in length using a linear variable displacement transducer (LVDT). A Pt-PtRh (Type R) thermocouple was attached to the center of the sample surface. The specimen temperature and the dilatometric data were measured simultaneously. Using the dilatometer, the dilatation curves of the specimens for the austenite-to-pearlite transformations was obtained during continuous cooling at various cooling rates. The conventional lever rule could be used to determine the phase fraction from the measured dilatation curves because carbon enrichment in austenite is not dominant during the pearlite transformation in high carbon steel (Suh et al., 2007a,b). The transformation plasticity was measured by supplying compressive stresses of 3, 5 and 8 MPa to support the specimen during the dilatometric experiment. Fig. 5 shows a schematic diagram of the specimen in the dilatometer.

A thermal infrared camera was used to measure the temperature distribution of the coil surface during cooling. The emissivity of the surface of the coil was 0.75. This data was used to determine the heat transfer coefficient at the coil surface by a minimization of the difference between the measured and calculated temperature profiles.

4. Results and discussion

Fig. 6 shows the measured and calculated temperature profile of the hot-rolled coil at 13 min from coiling. The calculation was based on bulk modeling. The measured data was obtained along the radial direction of the coil surface. The heat transfer coefficient at the coil surface was determined by minimizing the difference between the measured and calculated temperature profiles on the coil surface during cooling. From the optimization based on the FE calculation, the heat transfer coefficients at the inner and outer surface of the coil were determined to be 10 and 28 J/m² s K, respectively.

The material constants used to calculate the kinetics of the austenite-to-pearlite phase transition were determined by applying the dilatation data obtained during the continuous heat
treatments into the inverse additivity technique (Han and Park, 2001). The constants of Eq. (2) are listed in Table 1. Fig. 7 shows the measured and calculated phase transformation for pearlite behavior of the high carbon steel under various cooling conditions. The calculated results are in good agreement with the experimental data. The deformation behavior of the hot-rolled coil was simulated by implementing the mathematical description Eq. (2) of the phase transformation kinetics model in Table 1 into the FE model.

The transformation plasticity strain for the high carbon steel was measured from a dilatometric experiment under an external applied stress. Fig. 8 shows the temperature–strain curves obtained from the dilatometer during 0.5 °C/s cooling under compressive stress states of 3, 5 and 8 MPa. The transformation plasticity strain of the steel was quantified by a comparison of these curves. The constant $c_v$ in Eq. (5) could be adjusted using an optimization procedure, which was performed while changing the constants systematically until the sum of the squared differences between the experimental and calculated data reached a minimum. The optimum value of 0.3 was obtained for high carbon steel. The dilatation curves calculated for 3, 5 and 8 MPa were compared with the experimental data in Fig. 8. The transformation plasticity strain was in good agreement with the measured data. This optimum value of 0.3 for $c_v$ in Eq. (5) was used throughout the subsequent calculations.

Fig. 9(b) shows the calculated temperature history at some given points of the coil shown in Fig. 9(a) during natural cooling. The initial coiling temperature and strip thickness were set to 600 °C and 3.3 mm, respectively. At the initial stage of cooling, the temperature increased due to the heat generated by the phase transformation, as shown in Fig. 9(c). After the phase transformation, the temperature decreased as cooling proceeded. The phase transformation inside the coil was quite slow due to the higher temperature. However, the phase transformation progressed rapidly at the surface region after coiling.

Fig. 10 shows the calculated distribution of von-Mises stress in a hot-rolled coil at 4 h after cooling. The von-Mises stress developed as a result of gravity and the thermal contraction was concentrated at the contact region with the ground. The distribution and history of temperature and stress were used in the FE calculation for a single layered sheet.
Fig. 10. Distribution of von-Mises stress, which are calculated by bulk FE modeling, in hot-rolled coil at 4 h after coiling.

Fig. 11 gives an example of the calculated deformed shape of the hot coil after cooling to room temperature. The calculations were carried out with and without consideration of the transformation plasticity, respectively, based on the single layer FE modeling. The initial austenite fraction was assumed to be 100%. As shown in Fig. 11, the asymmetric contraction behavior of the coil during cooling could be reproduced successfully using the FE simulation considering the transformation plasticity. It was confirmed that the asymmetric contraction was caused by the small stress that develops naturally in the hot-rolled coil due to gravity. The ratio of the horizontal to perpendicular radius of the coil was used as a parameter to quantify the asymmetric contraction of the coil, and is denoted as the asymmetric contraction parameter.

One of the purposes of this study was to determine the effect of some selected process variables on the asymmetric contraction through a series of process simulations. First, the effect of the initial austenite fraction of the steel on the amount of asymmetric contraction was simulated. The initial coiling temperature and strip thickness were set to 600 °C and 3.3 mm, respectively. Fig. 12(a) shows the asymmetric contraction parameter with the variation of the initial austenite fraction. The amount of asymmetric contraction increased with increasing initial austenite fraction because the transformation plasticity strain increases with the phase transformed fraction, as shown in Eq. (5). Therefore, in order to avoid asymmetric contraction during cooling after coiling, it is necessary that the phase transformation is completed to the run-out table before coiling in the hot rolling process.

Fig. 12(b) shows the effect of the steel thickness on the level of asymmetric contraction. The initial coiling temperature and the initial austenite fraction were set to 600 °C and 0.5, respectively. Since the thicknesses of the oxide and interface layer were assumed to be constants for the entire calculation, the amount of asymmetric contraction increased with increasing steel thickness.
contraction of hot-rolled coil decreases with increasing steel thickness because the stiffness of steel is proportional to its thickness. This leads to the suggestion that an increase in tension force during coiling may play an important role in reducing the asymmetric contraction of the hot-rolled coil after coiling.

Fig. 12(c) shows the effect of the steel weight on unit area of the inner layer on the amount of asymmetric contraction. The initial coiling temperature and initial austenite fraction were set to 600 °C and 1.0, respectively. The decrease in weight per unit area of the inner layer decreases the amount of asymmetric contraction of the hot-rolled coil. This means that in an actual mill process, the total length of the coil is an important factor affecting the asymmetric contraction of a hot-rolled coil.

5. Conclusion

In conclusion, a finite element (FE) model incorporating transformation plasticity was used to analyze the thermo-mechanical and metallurgical behavior of hot-rolled steel during a phase transformation after coiling. The effect of the transformation plasticity on the asymmetric contraction of the hot-rolled coil was confirmed by the small amount of stress that developed naturally in the hot-rolled coil due to gravity. The FE simulations showed that the extent of the phase transformation before coiling, the tension force during coiling, and the steel weight per unit area of the inner layer in the coil are controllable process variables that can reduce the asymmetric contraction during cooling after the coiling of hot-rolled steel.

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References